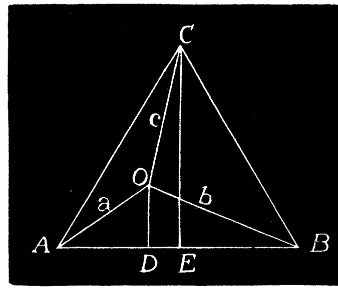
15. Proposed by SETH PRATT, C. E., Assyria, Michigan.

From a point in an equilateral triangle, the distances to the angles are, respectively, 20, 28, and 31 rods. Required a side of the triangle.

Solution by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

To establish a general formula, we denote the three given lines by a, b, c, and the required side by x.



Let the perpendicular $OD=z$, $AD=y$, $CE=\frac{x}{2}\sqrt{3}$, $AE=\frac{x}{2}$. Now, we obviously have the three equations: $y^2+z^2=a^2$....(1), $(x-y)^2+z^2=b^2$....(2), and $\left(\frac{x}{2}\sqrt{3}-z\right)^2+\left(\frac{x}{2}-y\right)^2=c^2$....(3).

Substituting the value of $y^2+z^2=a^2$ in (2), we get $x^2-2xy=b^2-a^2$, whence, $y=\dfrac{a^2-b^2+x^2}{2x}$....(4), and the value of $x^2+y^2+z^2$ from (2), in (3) and also (4) in (3), we obtain after a few easy reductions, $z=\dfrac{a^2+b^2-2c^2+x^2}{2x\sqrt{3}}$....(5). Substituting (4) and (5) in (1), we have without trouble the quadratic, $x^4-(a^2+b^2+c^2)x^2=a^2b^2+a^2c^2+b^2c^2-a^4-b^4-c^4$, whence

$x^2=\frac{1}{2}[a^2+b^2+c^2+\sqrt{3}\sqrt{2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4}]$ or if we denote the area of the triangle whose sides are a, b, c, by $\triangle$, we obtain the elegant expression, $x^2=\frac{1}{2}[a^2+b^2+c^2+4\triangle\sqrt{3}]$.

Substituting numerical values, we find $x^2=\frac{3}{2}[715+\sqrt{401557}]=44.97+$. The minus value of the radical must be rejected for the case that O is without the triangle.

Also solved in various ways by *P. S. BERG, J. W. WATSON, H. C. WHITAKER, R. H. YOUNG, G. B. ZERR, L. B. FRAKER*, and the *PROPOSER*.

16. Proposed by COLMAN BANCROFT, Professor of Mathematics, Hiram College, Hiram, Ohio.

A traveler whose speed constantly increases in a geometrical progression passes A at 2 o'clock, B at 3:30, C at 4:30, and D at 6:18. At B he is moving at the rate of 12 miles per hour, and at C 18 miles. Find his rate at A and D, and the distance from A to each of the points B, C, and D.

Solution by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.

Let a denote the rate at A, d the rate at D, r the rate of acceleration per hour. At the end of time t after 2 o'clock, the rate will be $a\left(1+\frac{r}{x}\right)^{tx}$, x being the number of times per hour the acceleration is added; this when x is infinite equals ae^{tr}; hence the equations $ae^{1.5r}=12$, $ae^{2.5r}=18$, $ae^{4.3r}=d$.

The distance$=\int_0^t ae^{tr}dt=\frac{a}{r}\left(e^{tr}-1\right)$.

Substituting for e its value 2.71828 and solving these equations,